Identification and Adaptation in Control Loops with Time Varying Gain

D. A. MELLICHAMP, D. R. COUGHANOWR, and L. B. KOPPEL

Purdue University, Lafayette, Indiana

A method is presented to apply the gain identification procedures developed in a companion paper to a control system. The method employs a small identification tank which follows the control tank; the identification tank is perturbed to estimate the gain of the process (controlled tank system). The effects of load changes on the identification system are minimized by this approach.

The results of analog and digital computer simulations of this adaptive process are given. Both a general system with linear change in gain and a pH control system with a step change in concentration of the buffer species are studied. Process gain changes up to 20:1 are introduced.

It is concluded that an adaptive control system of this type can be designed to maintain good control characteristics in a process experiencing wide gain variation. Criteria are presented to aid in the design of an adequate identifier.

All chemical processes exhibit nonstationary or time varying characteristics to some extent: flow variations influence system dynamics, catalyst efficiency drops off, measurement instruments do not hold their accuracy indefinitely, etc. The usual control approach is to consider the system as time-invariant and to design its controller with a safety factor sufficient to insure adequate operation. For example, we might use a gain margin of two in setting a particular process controller. This means that the process gain (change in output/change in input) could increase in time by a factor of two before the system would become unstable.

Some chemical processes exhibit very wide time variations in process characteristics. In reference *1* the *p*H control system was discussed; it was shown how the buffer level of a *p*H system inversely influences one important control characteristic, the process gain, which might vary easily by a factor of ten. Classical control techniques cannot be relied on with such systems. Even when process characteristics change slowly or when the changes are not severe in magnitude, classical techniques may lead to systems which are only partially satisfactory.

An alternate approach is to measure the time varying changes in the process and to make necessary modifications to the controller to correct for them. This is adaptive control. Reference *1* described a method to measure or identify the gain of a time varying flow process and illustrated the method with the *p*H problem. This paper discusses the application: construction of an adaptive control system to compensate for a time varying gain such as occurs in the *p*H system.

A method of characterizing or identifying the gain of a time varying process was presented in reference *1*. The results of a number of tests, both analytical and experimental, indicated that an identifier constructed with this identification algorithm possessed certain dynamics which can be expressed by

$$\hat{k}_I(t) = k_I\left(\frac{nT_I}{2} - \frac{T_I}{4}\right), n = 0, 1, \ldots \quad (1)$$

where

$$n = m \text{ for } \frac{mT_I}{2} \leq t < \frac{(m+1)T_I}{2}$$

$$m = 0, 1, \ldots$$

Hence the identifier estimate of the identification tank system gain is equivalent to sampling and holding (with period: $T_I/2$) the actual system gain, delayed by $T_I/4$.

In order for the identifier to exhibit such dynamics exactly, and not introduce scatter into the estimates, it is necessary that the system average output level $\bar{y}$ remain constant. In other words, such an identification method requires the system output variable (corresponding to stored energy) to be constant or, at least, requires low frequency fluctuations in $\bar{y}$ (with period $\sim T_I/2$) to be small compared to the output deviation amplitude. If this condition is satisfied, the identifier is termed *ideal*.

The maintenance of reasonably constant system output requires a regulating system incorporating feedback control, since the system is subject to unknown load disturbances. Uncontrolled load disturbances result in fluctuation in $\bar{y}$, which as indicated, is undesirable.

The direct application of an identification procedure of this type to a feedback control system is not possible. Any attempt to perturb a controlled process for identification purposes initiates compensating action from a properly designed controller, which sees the perturbance as a load disturbance. Thus, interaction between the control and identification loops in such a case would be too severe for the method to be practical.

However, as in the case of a *p*H system, the dynamics of certain flow systems are functionally related to the contents of the system. When such a relationship exists it is then possible to couple identification with a flow system under automatic control to effect controller adaptation;

D. A. Mellichamp is with E. I. du Pont de Nemours and Company, Inc., Kinston, North Carolina.

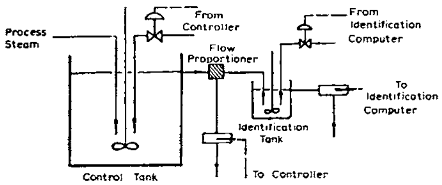

Fig. 1a. Simultaneous identification and control sequential arrangement.

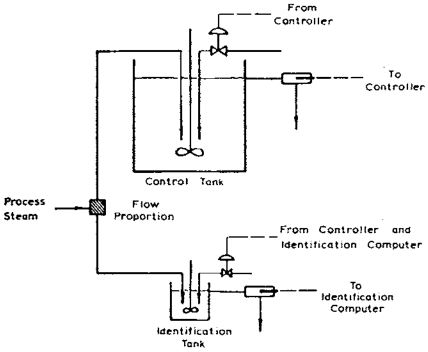

Fig. 1b. Simultaneous identification and control parallel arrangement.

that is, to adjust the controller gain to compensate for changes in the dynamic properties of the controlled system.

Figure 1 illustrates one possible approach. A small identification tank receives a portion of the effluent of the controlled process stream from the control tank. This identification tank receives material at or near the control set point (assuming that the controlled system is designed and functioning correctly) and the identification tank system is perturbed sinusoidally by varying an input stream. This might be cooling water flow rate, flow rate of some species, etc. Figure 1 indicates that this manipulated variable is the flow rate of a controlled stream. With gain varying systems, adaptive control is achieved by using the estimate of system gain obtained from the identifier to make compensating changes in the controller.

The disadvantages of such a procedure are the necessary duplication of equipment, and the necessity of driving the identification flow system output in both directions from the control set point. In the event that this latter consideration is a pertinent difficulty, alternate control–identification configurations may be available, for example a parallel arrangement of control and identification systems might be used.

In reference *1* the destabilizing influence of control loop gain variation was demonstrated by analyzing a *p*H regulating system. Theoretical consideration will now be given to gain varying processes of this type, where the emphasis will be on improving system response by means of adaptive procedures.

THEORETICAL

The simplest method to maintain good control system response in spite of a time varying process is to attempt to hold the overall control loop gain constant. From Equation (17) of reference *1* it can be seen that this policy maintains the characteristic form of the system response; an increasing or decreasing value of S in the numerator of Equation (17) does not modify its form, however.

If all the elements in the control loop are grouped into either the controller or the process, this policy is expressed by

$$k_c \, k_p = K_L \tag{2}$$

Therefore, the defining equation for the desired controller gain is

$$k_c|_{\text{desired}} = K_L/k_p \tag{3}$$

Since only an estimate of k_p is known, the simplest way to modify the controller equation is to use as the controller gain

$$k_c|_{\text{actual}} = K_L/\hat{k}_p \tag{4}$$

Because $\hat{k}_p$, as obtained by the identifier described earlier, is a staircase function, such a controlled gain relation is discontinuous. Techniques for proving system stability in nonautonomous systems are available [for example, Liapunov (*4*)]. However, none of these can be used conveniently with systems containing such a discontinuous parameter.

Experimental work indicates approximate stability limits for this control situation. Since we are seldom interested in stability alone, more emphasis is put on determining the limits for "good" control (in some subjective sense). These studies are not rigorous but indicate the general area of applicability of adaptive control systems as treated in this work. To assist in this it will be convenient to have a measure of the deviation of loop gain from the desired value K_L. The identification gain ratio (IGR) will be used as such a measure and is defined as follows:

$$\text{IGR} = \frac{\text{actual loop gain}}{\text{desired loop gain}} = \frac{k_c|_{\text{actual}}}{k_c|_{\text{desired}}} = \frac{k}{\hat{k}_p} \tag{5}$$

Ideally this quantity is equal to one.

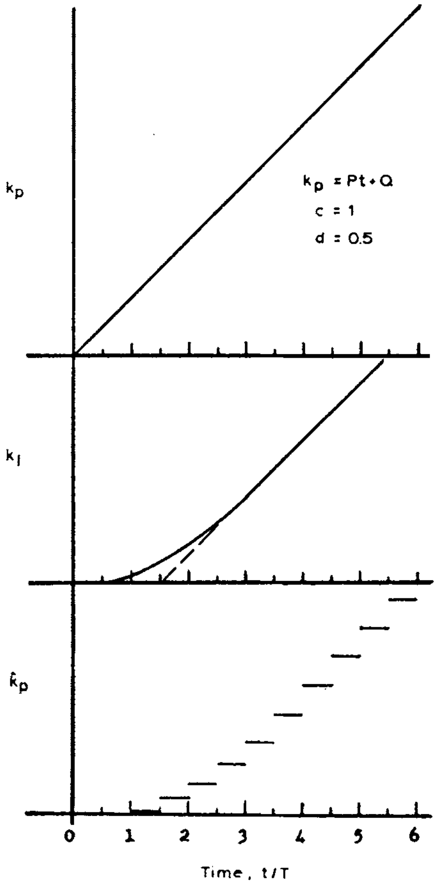

Fig. 2. Example of identification of a flow system with gain as a linear function of time.

Several specific systems with deterministic gain variations will now be considered for which the IGR may be predicted as a function of time.

GENERAL GAIN VARYING FLOW SYSTEM WITH THE GAIN AS A LINEAR FUNCTION OF TIME

If the gain of a particular stirred tank system can be directly related to some characteristic property of the fluid in the tank (for example, temperature, concentration of some species, etc.), a first-order differential equation relates the gain of the identification tank system to the gain of the process tank system

$$cT\frac{d(k_I)}{dt} + k_I = k_p\,(t - dT) \qquad (6)$$

It is now assumed that at time $t = 0$, some variation in the contents of the process tank (initially at steady state) occurs that changes the gain of this system linearly with time. The equation for the gain of this tank system is then

$$k_p = Pt + Q \qquad (7)$$

By using Equations (6) and (7), one can calculate the gain of the identification tank system as a function of time:

$$k_I = P\left[(t - dT) - cT\left(1 - \exp\left(-\frac{t - dT}{cT}\right)\right)\right] + Q \qquad (8)$$

where

$$t - dT = \begin{cases} 0, & t \leq dT \\ t - dT, & t > dT \end{cases}$$

If an ideal identifier is used to estimate the gain of this tank system, it is known that additional pure delay $cT/4$ and effective sampling and zero order holding with period $cT/2$ are introduced. The estimate of the identification tank gain (which is an estimate of the process gain) is

$$\hat{k}_I = \hat{k}_p = cTP\left\{\left(\frac{j}{2} - \frac{d}{c} - \frac{1}{4}\right) - \left[1 - \exp\left(-\frac{j}{2} + \frac{d}{c} + \frac{1}{4}\right)\right]\right\} + Q \quad (9)$$

where

$$\frac{j}{2} - \frac{d}{c} - \frac{1}{4} = \begin{cases} 0, & \frac{j}{2} \leq \frac{d}{c} + \frac{1}{4} \\ \frac{j}{2} - \frac{d}{c} - \frac{1}{4}, & \frac{j}{2} > \frac{d}{c} + \frac{1}{4} \end{cases} \qquad (10)$$

and j, the sample interval, takes on values 0, 1, ... Figure 2 illustrates an example of such a process.

For the process under consideration, the gain ratio of Equation (5) may be calculated from Equations (7) and (9)

$$\text{IGR} = \frac{\frac{P}{Q}t + 1}{\frac{cTp}{Q}\left\{\left(\frac{j}{2} - \frac{d}{c} - \frac{1}{4}\right) - \left[1 - \exp\left(-\frac{j}{2} + \frac{d}{c} + \frac{1}{4}\right)\right]\right\} + 1} \qquad (11)$$

where

$$j = m \text{ for } \frac{m}{2} \leq \frac{t}{cT} < \frac{m+1}{2}, m = 0, 1, \ldots$$

and Equation (10) must be satisfied.

If P is positive, that is, if the system gain is increasing, the gain ratio function goes through a series of local maxima which occur when $t = (j + 1)\,cT/2$. In addition, there is some extreme value of IGR which occurs for a particular integral value of j. This quantity will be useful in later performance studies and will be defined as

$$\text{ext (IGR)} = \max_{\{j\}}\left\{\frac{\frac{cTP}{Q}\frac{(j+1)}{2} + 1}{\frac{cTP}{Q}\left\{\left(\frac{j}{2} - \frac{d}{c} - \frac{1}{4}\right) - \left[1 - \exp\left(-\frac{j}{2} + \frac{d}{c} + \frac{1}{4}\right)\right]\right\} + 1}\right\} \qquad (12)$$

Ext (IGR) is a function of only two quantities: cTP/Q and d/c. The first is a normalized rate of change of process gain:

$$\frac{cTP}{Q} = \frac{d\left(\frac{k_p}{k_{pi}}\right)}{d\left(\frac{t}{cT}\right)} \qquad (13)$$

since

$$Q = k_p|_{\text{initial}} = k_{pi}$$

$$P = \frac{d(k_p)}{dt}$$

The quantity d/c is the ratio of transport dead time to identification tank time constant.

pH CONTROL FLOW SYSTEM WITH STEP CHANGE IN INLET STREAM CONCENTRATION OF A SINGLE BUFFER SPECIES

Consider the inlet stream L to a pH control process to contain only one weak acid and its residues $X' = x'$. In particular, if the phosphoric acid system is considered at or near $p\text{H} = 7$, the results of experimental work indicate that

$$k_p \propto S \cong S_o/x' \qquad (14)$$

where S_o is a constant.

Let x'_P and x'_I be the values of x' in the process and identification tanks, respectively. Let x'_i be the value of x' in the stream entering the process tank. A step change in x'_i will now be considered.

$$x'_i = \begin{cases} x'_{io}, & t < 0 \\ x'_{i\infty}, & t \geq 0 \end{cases} \qquad (15)$$

Let

$$M = x'_{i\infty} - x'_{io} \qquad (16)$$

x'_I and x'_P may be calculated readily by applying unsteady state material balances and by assuming that the dilution effects caused by the control reagents are negligible. The results are

$$x'_P = x'_{io} + M\,[1 - \exp(-t/T)] \qquad (17)$$

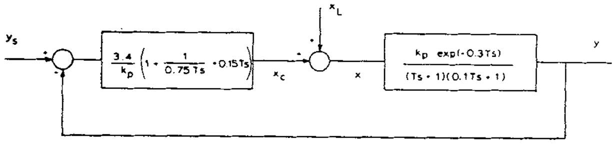

Fig. 3. Block diagram of controlled process with optimum three-mode controller settings.

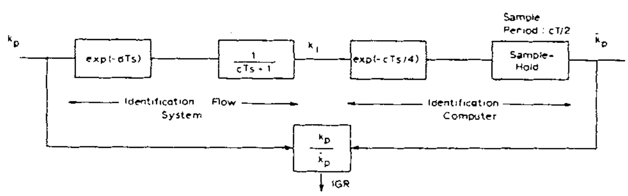

Fig. 4. Block diagram of identification simulation.

$$x'_I = x'_{io} + M\left\{1 - \frac{1}{1-c}\left[\exp\left(-\frac{t-dT}{T}\right) - c\exp\left(-\frac{t-dT}{cT}\right)\right]\right\} \quad (18)$$

$$t - dT = \begin{cases} 0 & t \leq dT \\ t-dT & t > dT \end{cases}$$

The gains in the two tank systems are

$$k_P(t) = S_o/x'_P(t)$$
$$k_I(t) = S_o/x'_I(t) \quad (19)$$

Now the estimate of k_I obtained by the identification computer is

$$\hat{k}_I(j) = k_I\left(\frac{jcT}{2} - \frac{cT}{4}\right) = \frac{S_o}{x'_I\left(\frac{jcT}{2} - \frac{cT}{4}\right)} \quad (20)$$

$$j = 0, 1, \ldots$$

where the delay and effective sampling introduced by the computer are indicated by the change of argument in going from Equation (19) to Equation (20).

The gain ratio function of Equation (5) can be written with Equations (17) and (20):

$$\text{IGR} = \frac{1 - D\left\{1 - \frac{1}{1-c}\left[\exp\left[-c\left(\frac{j}{2} - \frac{d}{c} - \frac{1}{4}\right)\right] - c\exp\left(-\frac{j}{2} + \frac{d}{c} + \frac{1}{4}\right)\right]\right\}}{1 - D\left[1 - \exp\left(-t/T\right)\right]} \quad (21)$$

$$D = \frac{x'_{io} - x'_{i\infty}}{x'_{io}}$$

$$j = m \text{ for } \frac{m}{2} < \frac{t}{T} \leq \frac{m+1}{2}$$

$$m = 0, 1, \ldots$$

and Equation (10) is satisfied. Equation (21) is written for a decreasing change in concentration of buffer ($D > 0$). This results in the more interesting case, IGR > 1, where the control loop gain becomes higher than the desired value K_L. In this case, the controlled system generally tends toward instability.

In a manner similar to that indicated previously, it can be shown that the extreme value of the gain ratio function is

$$\text{ext (IGR)} = \max_{\{j\}} \left| \frac{1 - D\left\{1 - \frac{1}{1-c}\left[\exp\left[-c\left(\frac{j}{2} - \frac{d}{c} - \frac{1}{4}\right)\right] - c\exp\left(-\frac{j}{2} + \frac{d}{c} + \frac{1}{4}\right)\right]\right\}}{1 - D\left[1 - \exp\left(-\frac{j+1}{2}\right)\right]} \right| \quad (22)$$

In this case ext (IGR) is a function of the three quantities: D, c, and d/c.

EXPERIMENTAL (COMPUTER STUDIES)

The experimental portions of this work consisted of a series of computer studies, both analog and digital, to explore the possibilities of control with process identification and compensation. Preliminary work was concerned first with the destabilizing effect of process gain variation in a typical control loop.

Many of the systems encountered in the process field can be adequately described by a differential equation with two time constants and a delay. Ordinarily, a three-mode (proportional–integral—derivative) controller would be available to control such a process. Latour (2) studied such systems extensively and presented optimum three-mode controller settings for such processes.

The system

$$G_p(s) = k_p\exp(-aTs)/(Ts+1)(bTs+1) \quad (23)$$

($a = 0.3$, $b = 0.1$) was chosen for the computer studies which follow. This could be a typical process that is somewhat difficult to control. Latour gave optimum controller settings: $K_L = 3.4$, $\tau_i = 0.75T$, $\tau_d = 0.15T$. The block diagram for this system is given in Figure 3 where provision for a load change has been made.

A number of tests were first made with the system (without variation in process gain) to determine the qualitative nature of the system load response for various values of loop gain (up to the point of instability). These tests were made to determine the gain at which the system just becomes unstable and the increase in loop gain (as a multiple of the optimum value, $K_L = 3.4$) which could still maintain a "good" process response. This latter determination was completely subjective but was based on the amount of oscillation in the response curve.

The results of these tests indicated instability at a loop gain of about 1.65 K_L (decay ratio > 1). Based on the amount of oscillation which might be tolerated, a loop gain of 1.2 $\times$ K_L is considered the limit of good control for this system.

GENERAL SYSTEM WITH GAIN AS A LINEAR FUNCTION OF TIME

A series of tests with an analog computer simulation of the controlled system described by Equation (23) was made to test the adaptive control techniques outlined previously. This system has the following closed-loop transfer function relating the output to load changes:

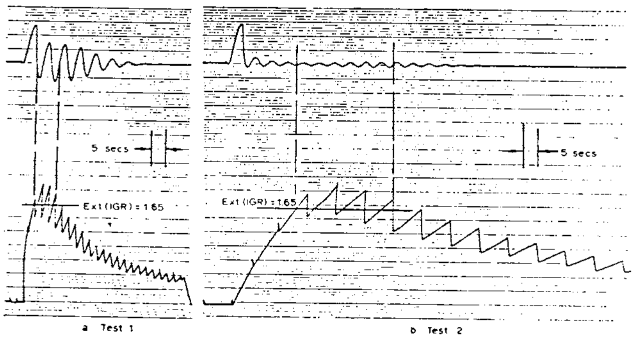

Fig. 5. Load response of a second-order system with changing gain.

$$\frac{y(s)}{x_L(s)} = \frac{k_P\,G_P(s)}{1 + K_L\,G_c(s)\,G_p(s)} \tag{24}$$

The computer simulation which was used in these studies eliminated the effect of k_P in the numerator of Equation (24). Since it was desired to study apparent stability of the system by examining the nature of its response to a load upset (to see whether the ultimate periodic component grew or decayed), the magnitude change in the output response caused by change in k_P was not desired.

Again it was assumed that the process is some sort of flow system and that a small identification tank (time constant: cT), which receives a portion of the process effluent, is used. The effect of a linearly changing process gain on identification and control was investigated with the further assumption that an ideal identifier is used to estimate the gain of the identification tank. (A model of the identification computer was used rather than the computer itself, because of the decreased computational difficulty.)

Figure 4 illustrates the block diagram approximation of the identifier. The identification flow system which was simulated is described by Equation (6).

Preliminary theoretical work showed that the IGR quantity is a function only of the dimensionless groups cTP/Q and d/c for the process. Figure 5 illustrates the IGR curves for two tests, each with $cTP/Q = 0.5$ and $d/c = 0.5$. In test 1, $T = 5$, $P = 0.5$, $c = 1$, $d = 0.5$, in test 2, $T = 5$, $P = 0.5$, $c = 4$, $d = 2$. As expected, the sequence of peak values is the same; however, the time scaling is different.

As part of these two tests, the system response to a step change in load was observed. The load change was applied at the instant the linear change in process gain was initiated. Note that the response curves shown in Figure 5 become more oscillatory as the loop gain increases. The interesting point is that during the time the IGR function is greater than 1.65, the instantaneous decay ratio is greater than one. Hence it is elected to define the system as temporarily unstable. (If it were applicable, linear theory would predict an unstable system with this situation, since the gain margin would be less than one.)

In tests 1 and 2 the controlled system is extremely fast with respect to the identification system ($c \geqq 1$). This situation is very unrealistic but the tests illustrate the property of "temporary instability." Further tests with the computer indicated that, if the process response is fast with respect to identification, the controlled system response is no worse than is indicated by the extreme value of the IGR function (for example, if ext (IGR) = 1.50, the controlled system response may temporarily exhibit the oscillatory nature of a system with loop gain equal to 1.5 times the optimum value, but no worse).

With faster identification ($c \leqq 0.2$), the controlled system load response never appeared as oscillatory as the ext (IGR) value would indicate. This is the more realistic case so that the quantity ext (IGR) can be used as a generally conservative estimate of the expected nature of the system response. For this system studied, the desired condition is ext (IGR) $\leqq 1.20$.

Results of these tests indicated that the quantity ext (IGR), as defined by Equation (12), can be used as a indication of short-term system stability. It is most useful in predicting if adequate control will be obtained for a given choice of system gain characteristics (P, Q) and identification system characteristics (c, d). A digital computer program for the IBM 7090 was written to calculate the IGR function given by Equation (11) for a number of combinations of cTP/Q and d/c. The ext (IGR) values for each pair cTP/Q and d/c were obtained from these results. Figure 6 gives a plot of ext (IGR) vs. cTP/Q for lines of constant d/c. This graph can be used to predict "adequate control" for any combination of identification system parameters and linear system gain change. With this approach, the selection of the proper identification system (c, d) for a process which is subjected to a linear variation in gain (P, Q) can be made with only a knowledge of the deviation (that can be tolerated by the controlled system) between the actual control loop gain and the desired value of loop gain.

The experience of this work would indicate that if the ext (IGR) function is kept at or below this value, the system response will be acceptable. In practice this means that Figure 6, showing ext (IGR) as a function of cTP/Q and d/c, can be used with any control system with which such an identification method is practicable.

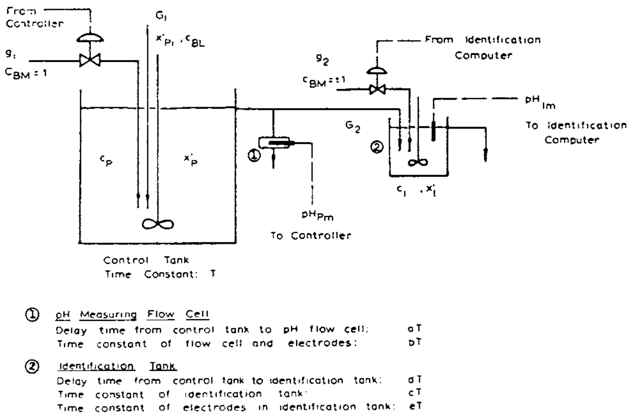

Fig. 7. Identification of process gain in a *pH* control system.

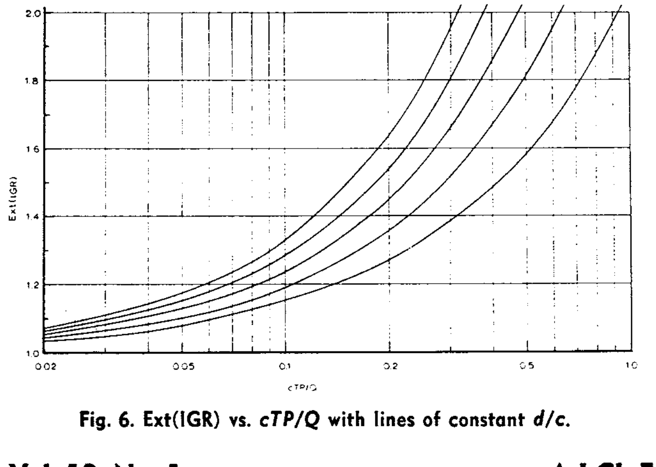

Fig. 6. Ext(IGR) vs. *cTP/Q* with lines of constant *d/c*.

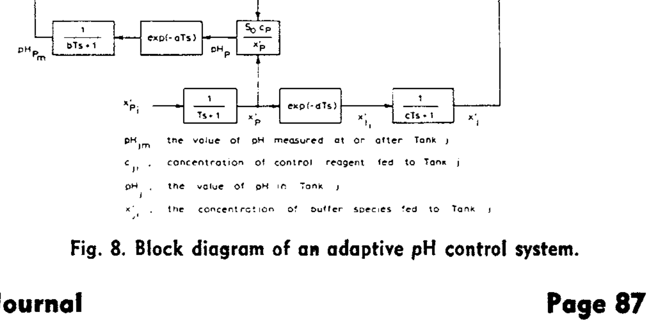

Fig. 8. Block diagram of an adaptive *pH* control system.

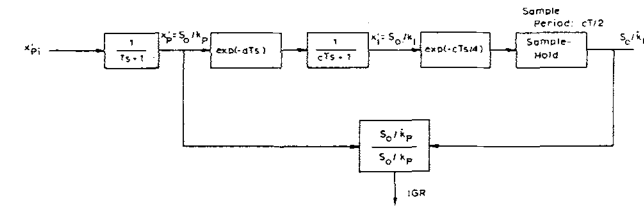

Fig. 9. Block diagram of pH process gain change and simulated identifier.

Note that a similar analysis could be made for decreasing system gain. Here the problem would be to avoid a system which is too sluggish.

pH Control System with a Step Change in the Concentration of a Single Buffer Species Present in the Feed Stream

In this section the more specific system outlined earlied (a pH regulating process) will be considered. Figure 7 illustrates the configuration for adaptive control of a pH regulating system. As indicated in the sketch, the perturbing reagent to the identification tank can take on normalized concentration values of either plus or minus one, that is, both acid and base of equivalent normality are available. The flow rates g_1 and g_2 are small compared to the other streams entering the respective tanks. Only one buffer species in the load stream will be considered, and it will be assumed also that the relation between S and x' is given by Equation (14). The deviation in pH is then related to deviation in the control reagent concentration by the expression

$$pH_j^* = \frac{S_o c_j^*}{x_j'} \tag{25}$$

where j refers to any specific portion of the process.

A block diagram which describes this process is given in Figure 8 where all the variables are in deviation form except the x_j'. The incorporation of an identification computer and gain adjustable process controller is indicated.

An analog simulation of this process, similar to that described in the previous section, was used to investigate control under conditions of varying gain. Again, the identifier was approximated as pure delay plus sampler. Figure 9 shows the block diagram of the simulated identifier.

Again it was found that the quantity ext (IGR) is useful as a measure of the deviation of controlled system response from the optimum response. This is the result of the control loop gain changing from the optimum value. The results of test 8 are given as an example where the controlled system again has the characteristics $T = 50$, $a = 0.3$, $b = 0.1$. For this test $D = 0.9$ (tenfold ultimate change in system gain), $d = 0.1$, $c = 0.1$ were selected. Figure 10 presents the strip chart recordings of this test. Figure 10a shows the optimum system response for comparison. Figure 10b gives the system response and a plot of the IGR function which result from a simultaneous application of step changes in load and in the entering buffer concentration x'.

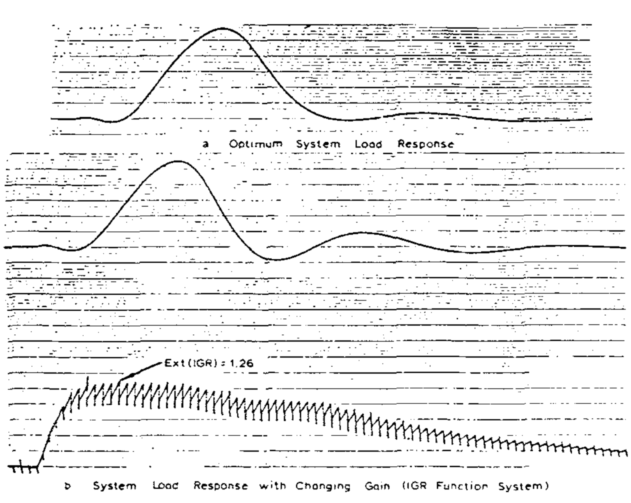

Fig. 10. Load response of pH control system with changing gain (test 8).

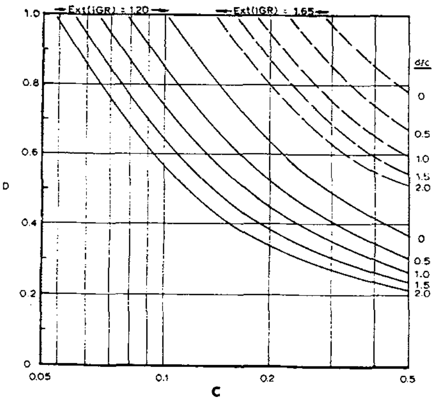

Fig. 11. Ext(IGR) contour chart, solution to Equation (22).

With this pH control system it is noted that the IGR function does not reach its extreme value and drop off rapidly, as was the case with the general second-order system with linear system gain change. As a result, the system response was almost as oscillatory as the curve shown in Figure 4 with a loop gain of 1.26 K_L. This is the value of ext (IGR) obtained momentarily in this test.

Again, to assist in predicting the characteristics of an adaptive system operating in this fashion, the IGR functions for various combinations of D, c, and d/c were obtained with a digital computer solution of Equation (21). Figure 11 gives a summary of the results: a typical contour plot with values of ext (IGR) of 1.20 and 1.65. Reference 3 contains more detailed design data.

As before, the specific controlled system is analyzed to determine the amount of deviation from the desired loop gain which can occur and yet still have adequate control. This quantity is taken as the tolerable extreme value of the IGR function. Figure 11 then can be used to specify the maximum allowable buffer level change (D) with a given identification system (c, d) or, alternatively, the identification system design parameters can be obtained if D is known.

In the event that sluggish system response is a problem, a similar approach can be used for a decreasing system gain $D < 0$.

Analog Simulation of the Adaptive pH Control System with the Actual Identification Computer

Verification of Results Obtained with the Identifier Model. The control studies presented up to this point all utilized the so-called ideal identifier characteristics. These ideal characteristics were used to derive approximate analytical expressions for the estimated process gain and to facilitate simulation of the control problem on the analog computer. This final study incorporated the entire adaptive pH control scheme as presented in Figure 8, including the actual identification computer. This was done to test the assumption that the identifier could be approximated by pure delay plus sampler. In addition, the effect of controlled system output variation on identification was studied.

To do this, a simulation of a pH control tank system was built on the analog computer. Coupled with this was a simulation of the identification tank system.

To test the results obtained from the digital computer solutions of Equation (21), three cases were studied:

	Test 1	Test 2	Test 3
D	0.9	0.9	0.9
C	0.2	0.2	0.5
d/c	0.5	0.0	0.0

With the controlled system at steady state, the step change in buffer level was made. Both the system gain and the identifier estimate of system gain were recorded. A summary of the results of the analog computer simulations is given in the following table.

Test No.	D	c	d/c	Ext (IGR) Analog	Ext (IGR) Digital
1	0.9	0.2	0.5	1.50	1.46
2	0.9	0.2	0.0	1.38	1.34
3	0.9	0.5	0.0	1.99	1.96

The important characteristic in each test was the ext (IGR) value. The values computed for these three tests are given and for comparison the values found by digital computer solution of Equation (21) are shown.

Figure 12 presents the x-y plotter recording of test 3. The curve representing the process gain was calculated directly from the x_P' curve, since k_P was not computed as an explicit function in the computer circuits. The significant lag between the process and the identifier here is the result of the high value of c. The poor estimation of process characteristics in this case is mirrored in the high value of ext (IGR).

The results of these tests are in good agreement with those obtained with the model of the identifier as shown by the close agreement between the ext (IGR) values in the above table. The discrepancies are within the estimated accuracy of the model which was used and the limits of accuracy of the analog computer. It is felt that the conclusions obtained using the modeled identifier are accurate. Hence the ext (IGR) contour curves in Figures 8 and 11, which were obtained with a model of the identifier, are applicable for the actual identifier.

It should be noted that the process gain curve in these simulations of the pH control system can be approximated by a linear change. Alternatively, the slope of this approximating straight line $\left(\text{normalized to obtain the form } \frac{d(k_P)}{d(Q)} \Big/ \frac{d(t)}{d(cT)}\right)$ can be used with the ext (IGR) curve (Figure 8) obtained for the general second-order system. This approach also gives a satisfactory estimate of the efficiency of identification as measured by the IGR function.

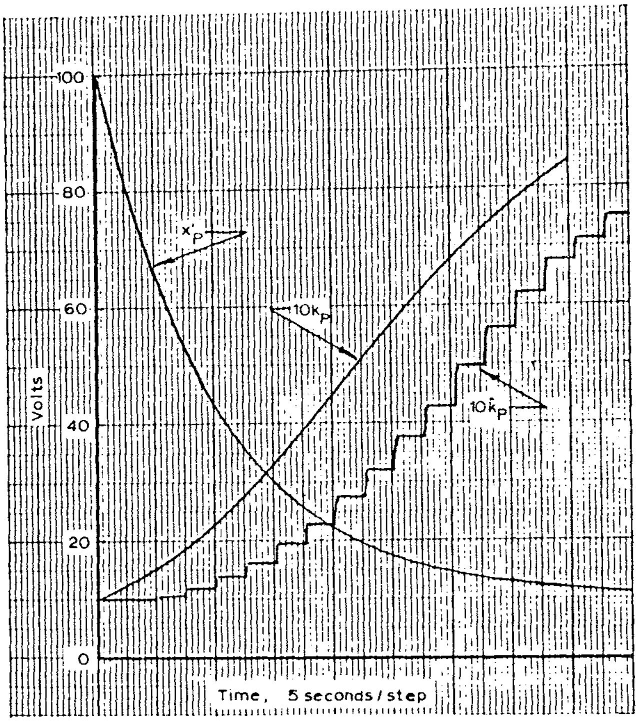

Fig. 12. Identifier response to changing system gain in pH control system (test 3).

CONCLUSIONS

1. In the case of a lumped parameter flow system which has a time varying gain functionally related to the material in the system at any time, the identification tank system should be operated separately from the control system to identify process gain accurately in spite of unknown load changes.

2. The problem of designing the identification system can be considered in terms of the expected variation in controlled process gain and the variation in control loop gain which can be tolerated. An identification–control adaptive system can be designed to maintain control loop gain within these limits, thus maintaining good control characteristics.

3. The major cause of nonideal identification (variation in identifier estimates of process gain) is variation in the output operating level of the identification flow system. This variation can be minimized by proper design of the control system. The identification–control system interaction can be minimized by employing as large a magnitude of input perturbing signal as is possible. Also, to decrease further this effect, feedforward techniques may be applicable. Smoothing and prediction techniques also are implemented easily on the computer. With real control systems of an especially difficult nature, such filtering methods may be required despite the increased identification lag which is introduced.

NOTATION

c = ratio of identification tank time constant to process tank time constant
D = dimensionless magnitude of the change in x'_i defined in Equation (21)
d = ratio of delay time to process tank time constant
$G_p(s)$ = transfer function of controlled process, defined by Equation (23)
IGR = identification gain ratio defined by Equation (5)
K_L = desired value of loop gain
k_c = controller gain
k_I = identification system gain
k_p = controlled process gain
M = magnitude of the step change in x'_i defined by Equation (16)
P = rate of change of system gain
Q = initial steady state value (positive)
S_o = constant in the relationship between k_p and x' of Equation (14)
T = basic process tank time constant
T_I = basic time constant of the identification flow system
τ_i = three-mode controller integral time constant
τ_d = three-mode controller derivative time constant

Subscripts

I = property of identification system
i = property of the inlet stream
T = property of the process tank
o = condition at time 0

LITERATURE CITED

1. Mellichamp, D. A., D. R. Coughanowr, and L. B. Koppel, *A.I.Ch.E. J.*, 12, No. 1, 75 (1966).
2. Latour, P. R., MS. thesis, Purdue Univ., Lafayette, Ind. (1964).
3. Mellichamp, D. A., Ph.D. thesis, Purdue Univ., Lafayette, Ind. (1964).
4. Struble, R. A., "Nonlinear Differential Equations," pp. 160-167, McGraw-Hill, New York (1962).

Manuscript received November 9, 1964; revision received June 21, 1965; paper accepted July 23, 1965.